Algorithms for Data Structures: Heuristic Search

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Aims

- Once you’ve understood this you should be able to:
  - Explain the idea of a heuristic
  - Devise simple heuristics
  - Carry out best-first search, hill climbing and A* search

Heuristics

- So far we've looked at strategies for searching when we know very little about the problem
- Heuristics are rules of thumb:
  - Approximate
  - Quick to compute
  - Not guaranteed to work
- Informed (or heuristic) search uses rules of thumb to guide search and cut down the amount of work we have to do
- Heuristics are used throughout AI
- We will go through a heuristic estimate of the distance (or cost) between the current state and the goal state

Example Heuristic: Estimate of distance to go

- Consider the 8-puzzle tile sliding game:
  - Goal state:

1	2	3
8		4
7	6	5

  - Which of the following is closer to the goal?

State A

2	8	3
1	6	4
7		5

or

State B

1	2	3
8	6	4
7	5	

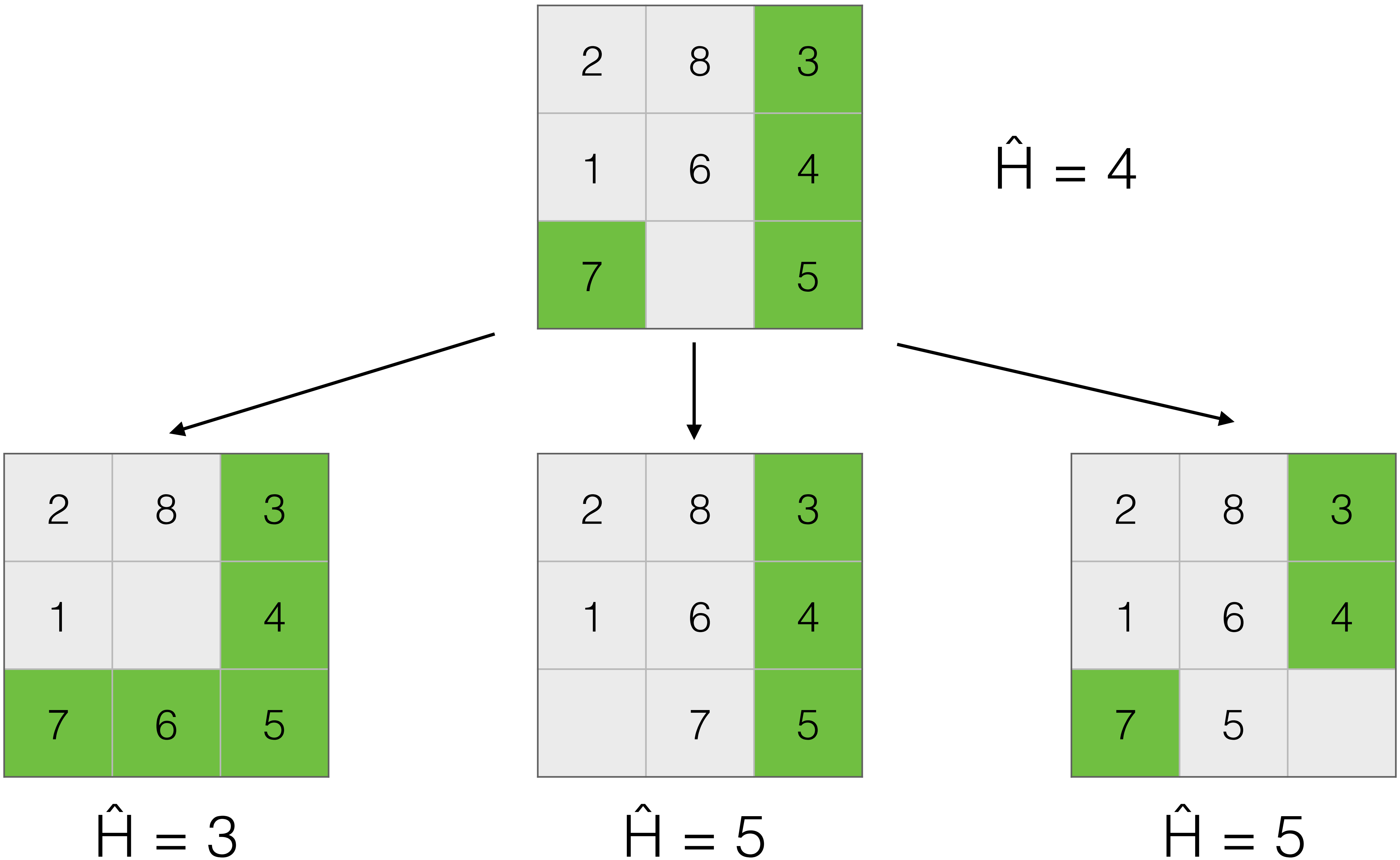
?

- One heuristic is to count the number of tiles our of place:
  - $\hat{H}(A) = 4$ $\hat{H}(B) = 2$
  - $\hat{H}$ is our heuristic estimate of the actual number of moves left
  - $H(A) = 5$ $H(B) = 2$

Hill Climbing

- How can we use our heuristic estimate of the distance to a goal state?
- In steepest-ascent hill climbing we generate the children of the current state
- We calculate the heuristic value of each
- Then select the one with the ‘best’ heuristic value
- Repeat until you can’t improve

Hill Climbing Example



Hill Climbing Gets Stuck

- Often hill climbing will reach a point where it can't improve further:

2	8	3
1		4
7	6	5

$\hat{H} = 3$

- This is an example of a plateau

- There is no efficient way to cross a large plateau if there is (by definition) no information to guide the search

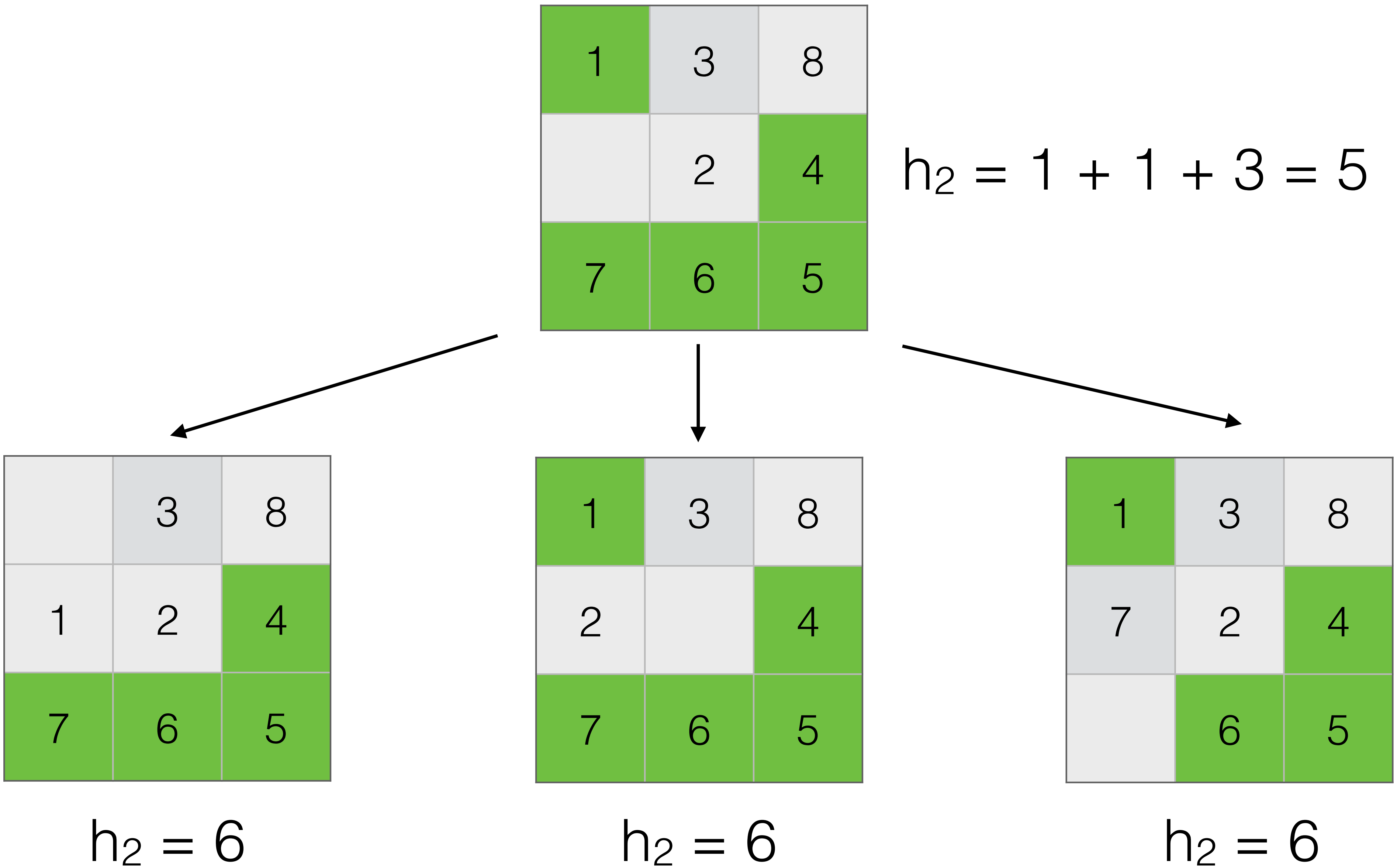
Hill Climbing Gets Stuck

- Hill climbing can also get stuck on local maxima (or minima if we're doing gradient descent)

- We can see this in the 8-puzzle example if we change the heuristic:

- Heuristic 2 h_2: for each tile add its vertical and horizontal displacement from its desired position. Sum these values across all the tiles.

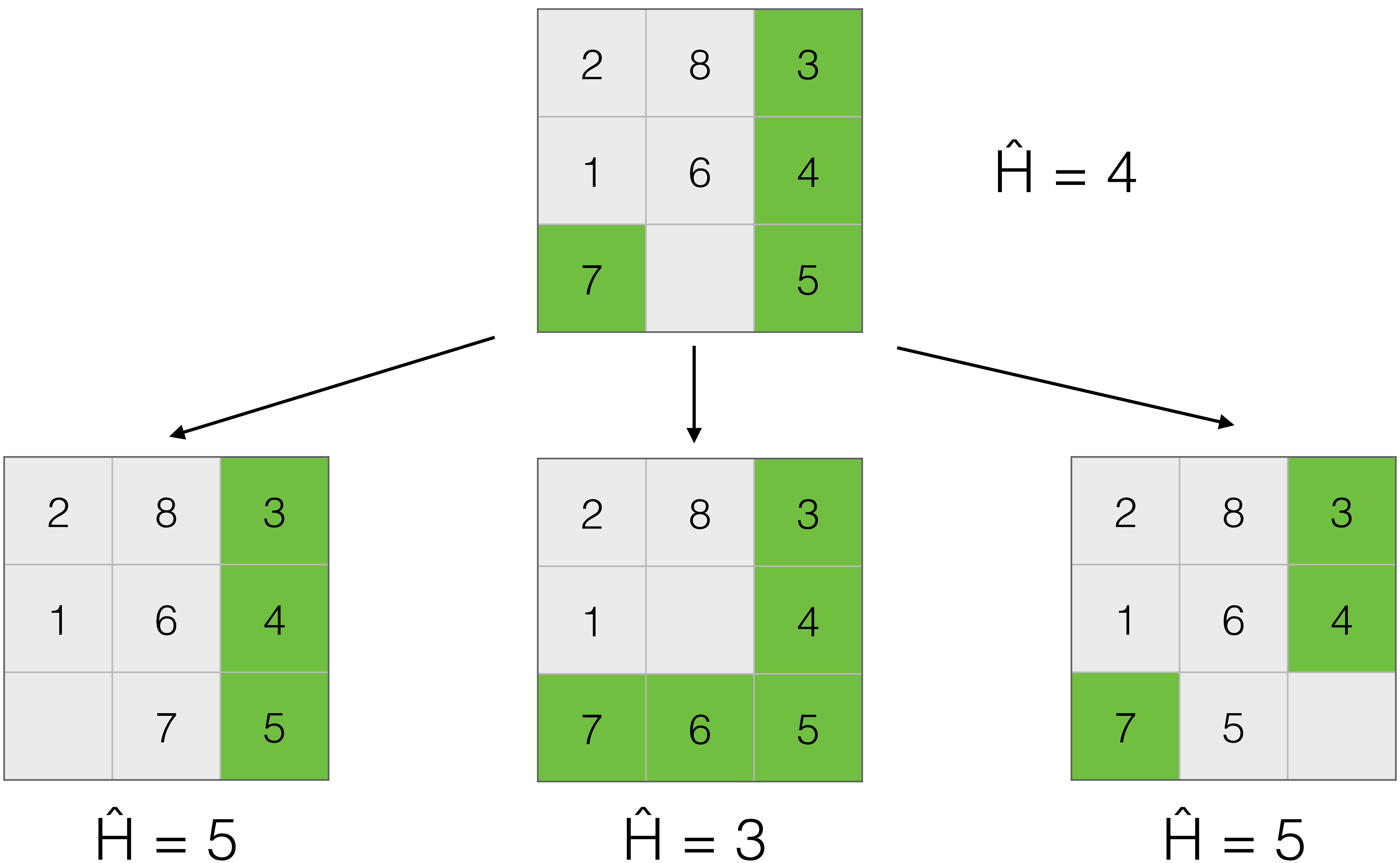
Hill Climbing Gets Stuck



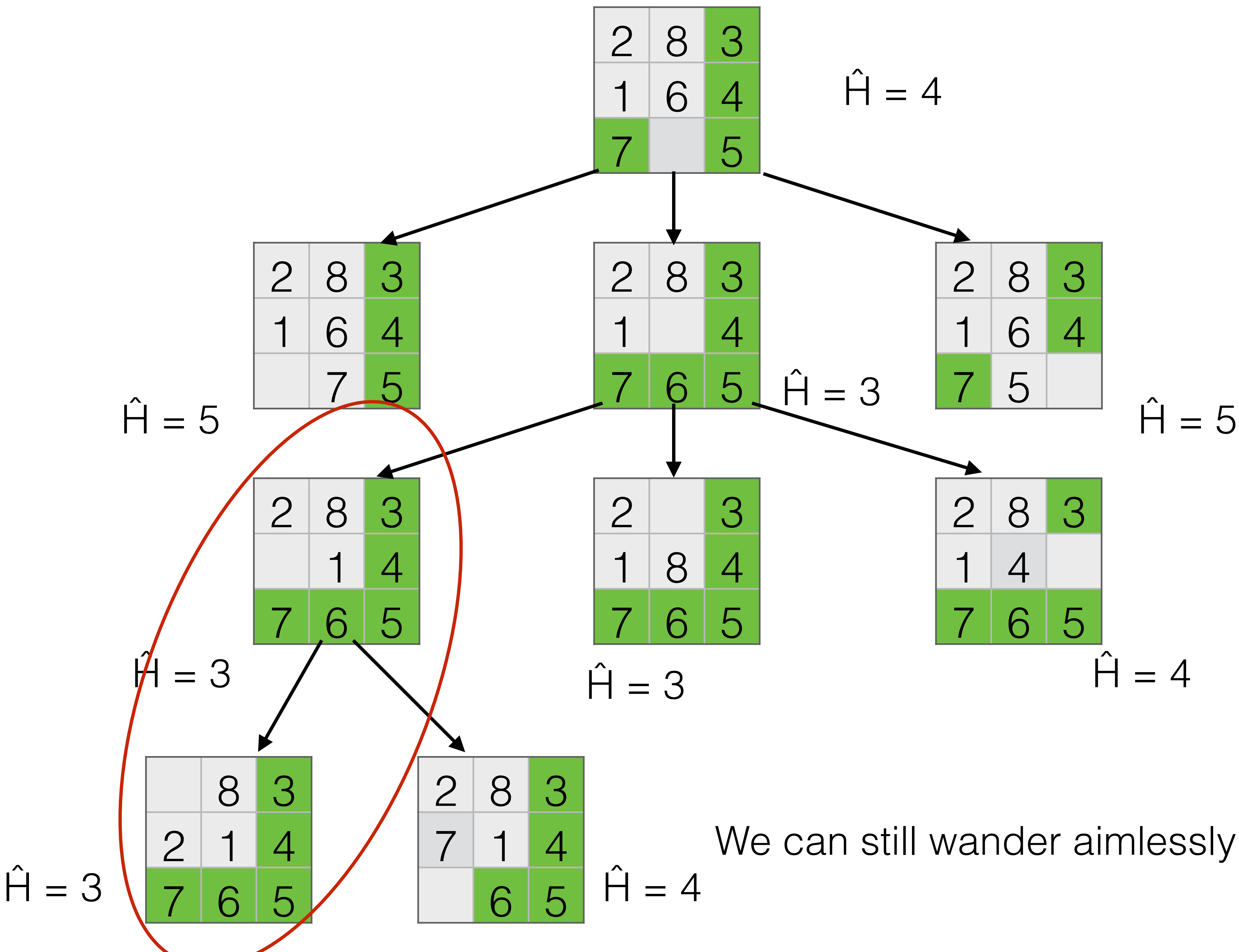
Best-First Search

- Remember the complete search tree you've explored so far (as in breadth-first search)
- But use $\hat{H}$ (evaluation function) to decide which leaf node to expand next, instead of path cost
- A venerable, but inaccurate name
  - If we really could choose the best node to expand, then it wouldn't really be a search at all
  - All we can do is choose the 'best' according to an evaluation function

Best-First Search



Best-First Search



We can still wander aimlessly however…

A* Search

- To obtain better searching we need to take into account the cost of the path <u>so far</u>

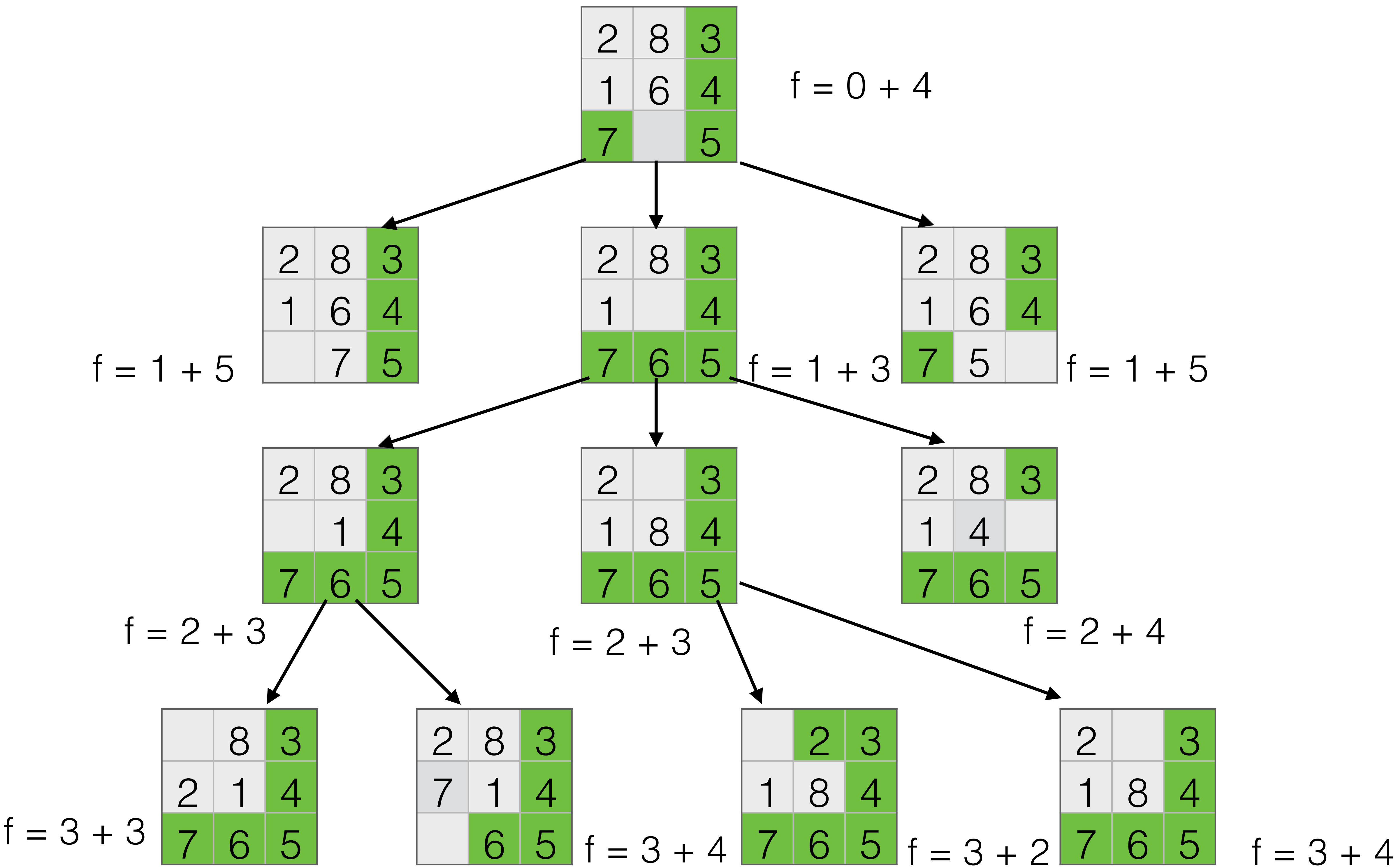
- $g(A)$ = cost (length) of the path from the root node to node A

- $\hat{H}(A)$ = heuristic estimate of the cost (length) of the path from node A to a goal state

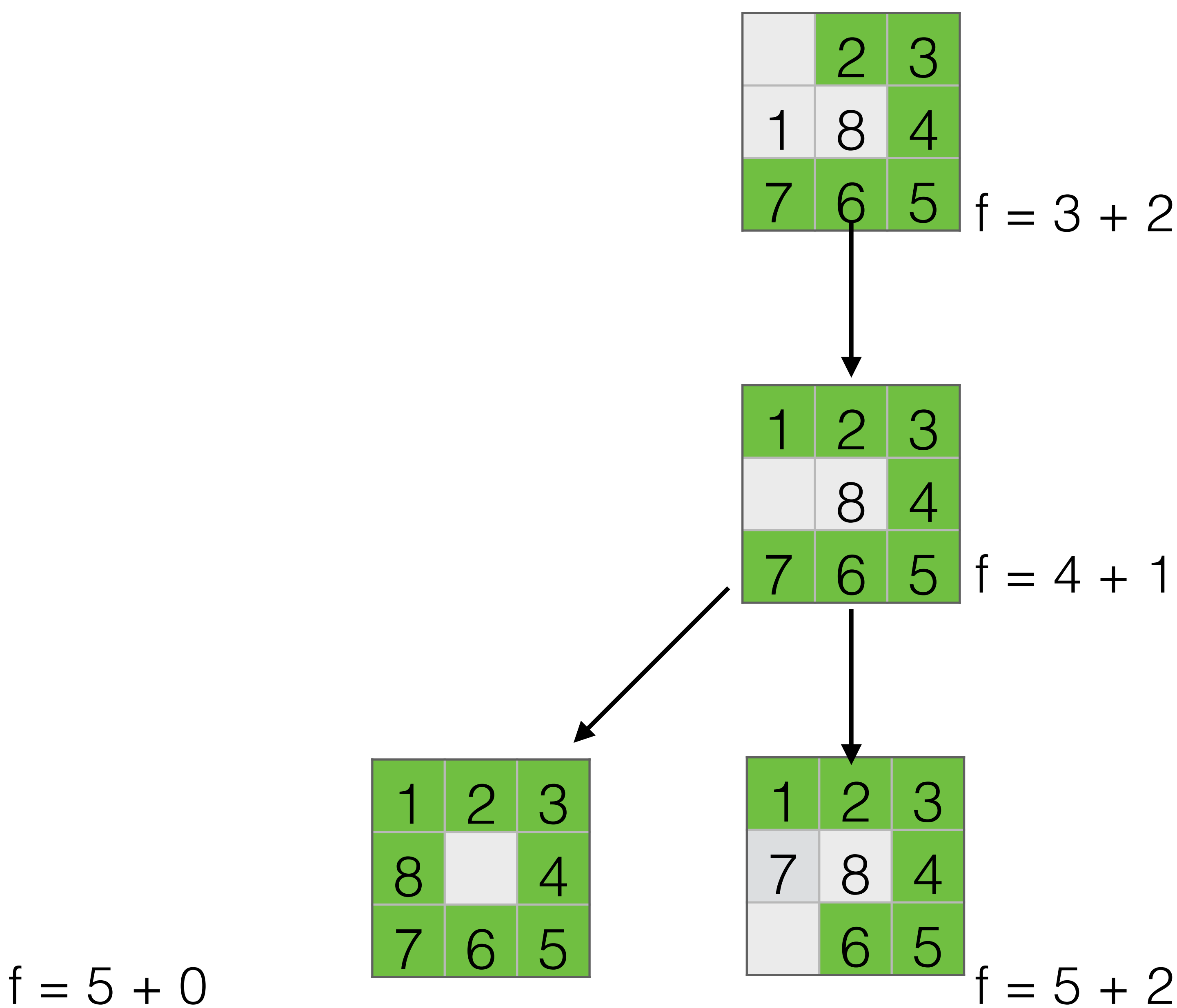
- $f(A) = g(A) + \hat{H}(A)$

- $f(A)$ is an <u>estimate</u> of the total cost of the path through A that starts at the root node and ends in the goal node

A* Search: Example



A* Search: Example



GOAL, woohoo!

Inventing Heuristics

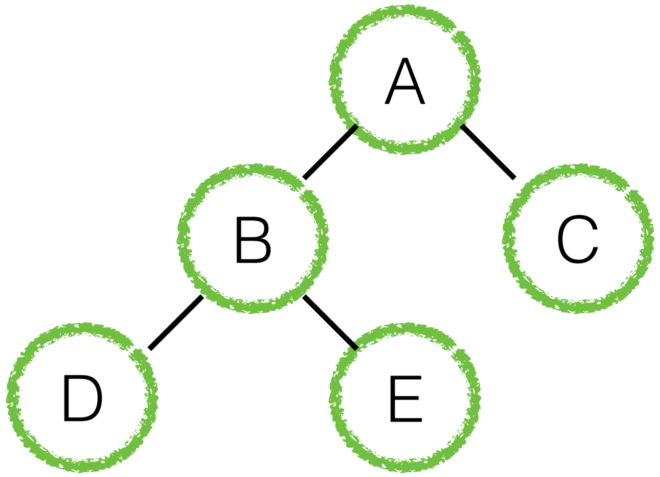
- $\hat{H}$ and h_2 are fairly good heuristics, but how do we invent one which is possibly better?
- Is it possible for a machine to create such a heuristic?
- Composite heuristic
  - Uses whichever currently defined heuristic returns the best result
- Statistical information:
  - Run our search 100 times and examine patterns
  - When $h_2(n) = 14$, it turns out that 90% of the time the real distance to the goal is 18. We can therefore us 18 as the real value when 14 is returned

Search: the story so far…

- We've seen:
  - depth-first (depth-limited, DFID)
  - breadth-first
  - best-first with $\hat{H}$
  - best-first with f (A* search)
- We can unify all these (mostly) into a single framework
- We can do this using the <u>idea of an agenda</u>

Agenda Based Search

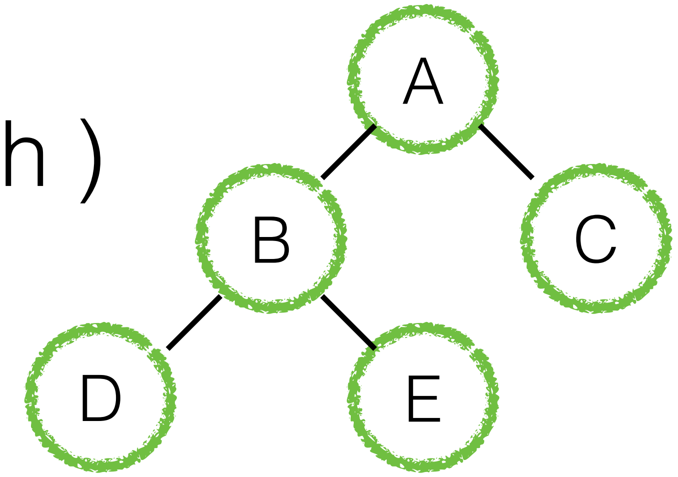
- In all our algorithms we have to choose which leaf node in the search tree to expand



- We can split the nodes into two lists:
  - OPEN = [D E C] - nodes to expand (leaf)
  - CLOSED = [A B] - nodes already expanded (internal)

Agenda Based Search

- Suppose we reorder the nodes in OPEN according to some criterion?
  - e.g reorder by depth of node in tree
    - deepest first (depth-first search)
      - OPEN = [D E C]
    - shallowest first (breadth-first search)
      - OPEN = [C D E]



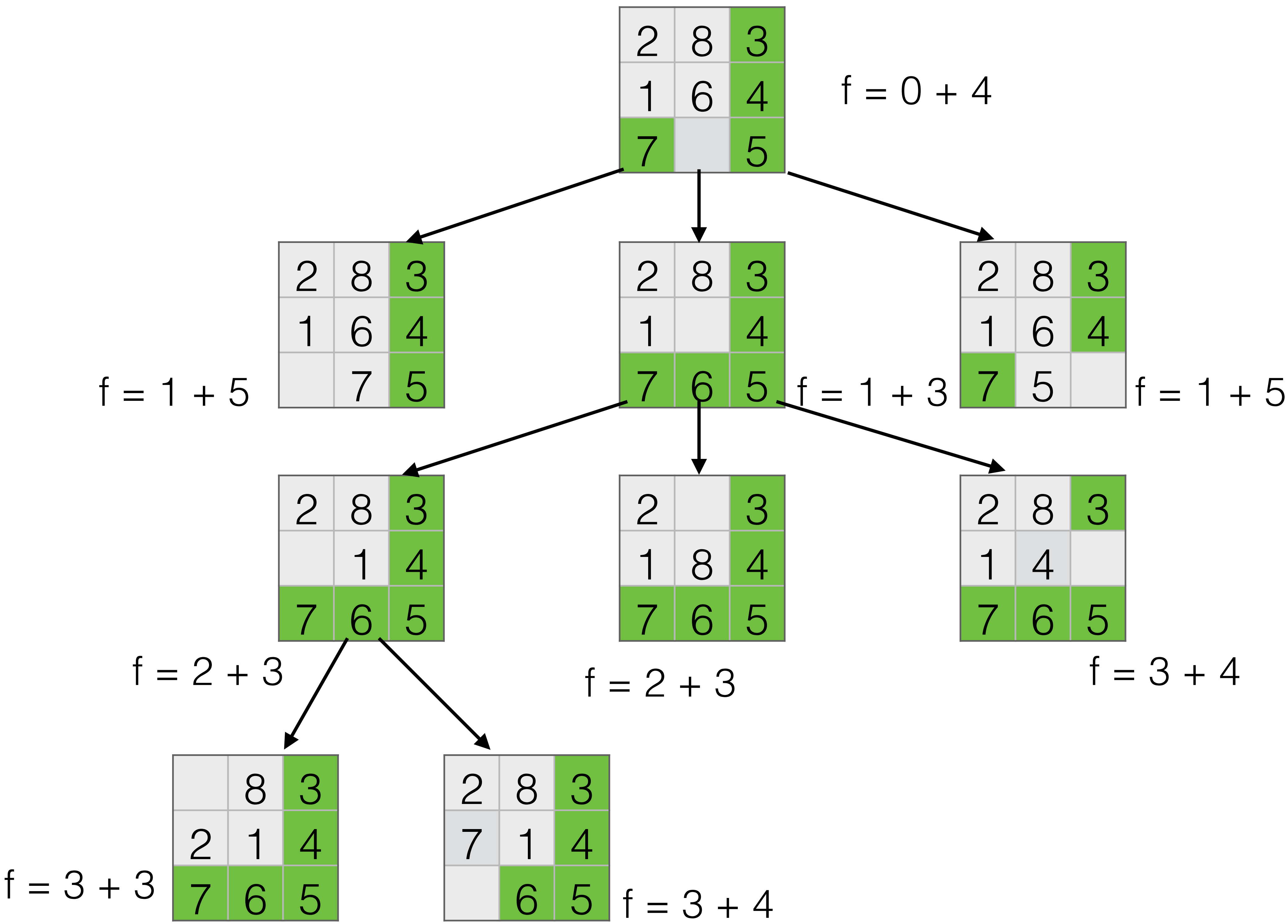
Agenda Based Search

- We then:
  - Expand the first node in OPEN
  - put it in CLOSED
  - put its children in OPEN
  - reorder OPEN
  - (NB to obtain depth-first search we also need to delete nodes from CLOSED when we backtrack)

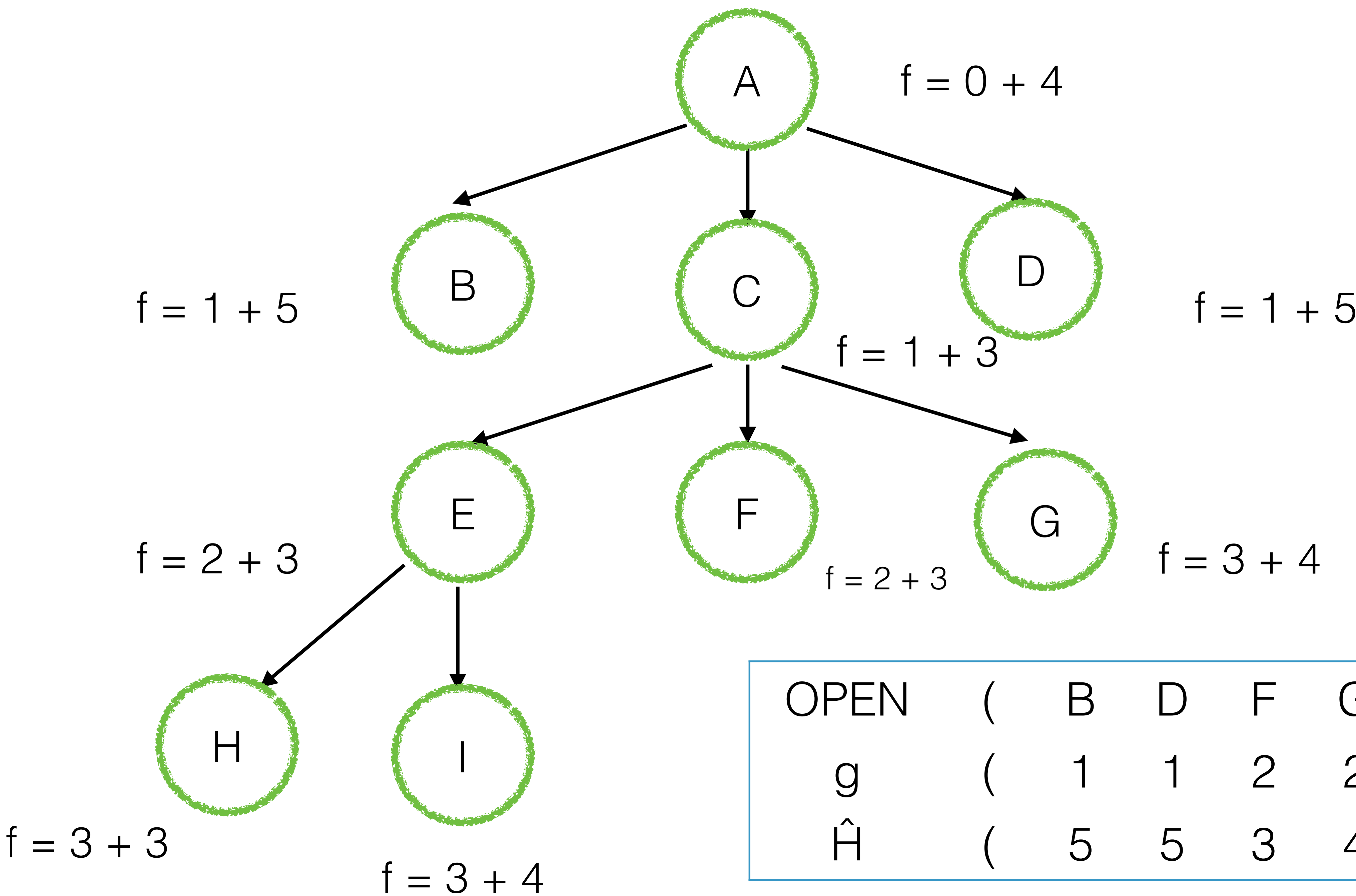
Agenda Based Search

- We can also implement best-first search in this way
- If we reorder OPEN by $\hat{H}$ then we have best-first search as described in the last lecture
- This is actually called greedy search
- Best-first search using $\hat{H}$ to reorder OPEN = greedy search
- Best-first search using g to reorder OPEN = uniform cost search
- Best-first search using $f = g + \hat{H}$ to reorder OPEN = A* search
- NB if g is just the depth of the node in the tree then uniform cost search = breadth-first search

Agenda Based Search: Example

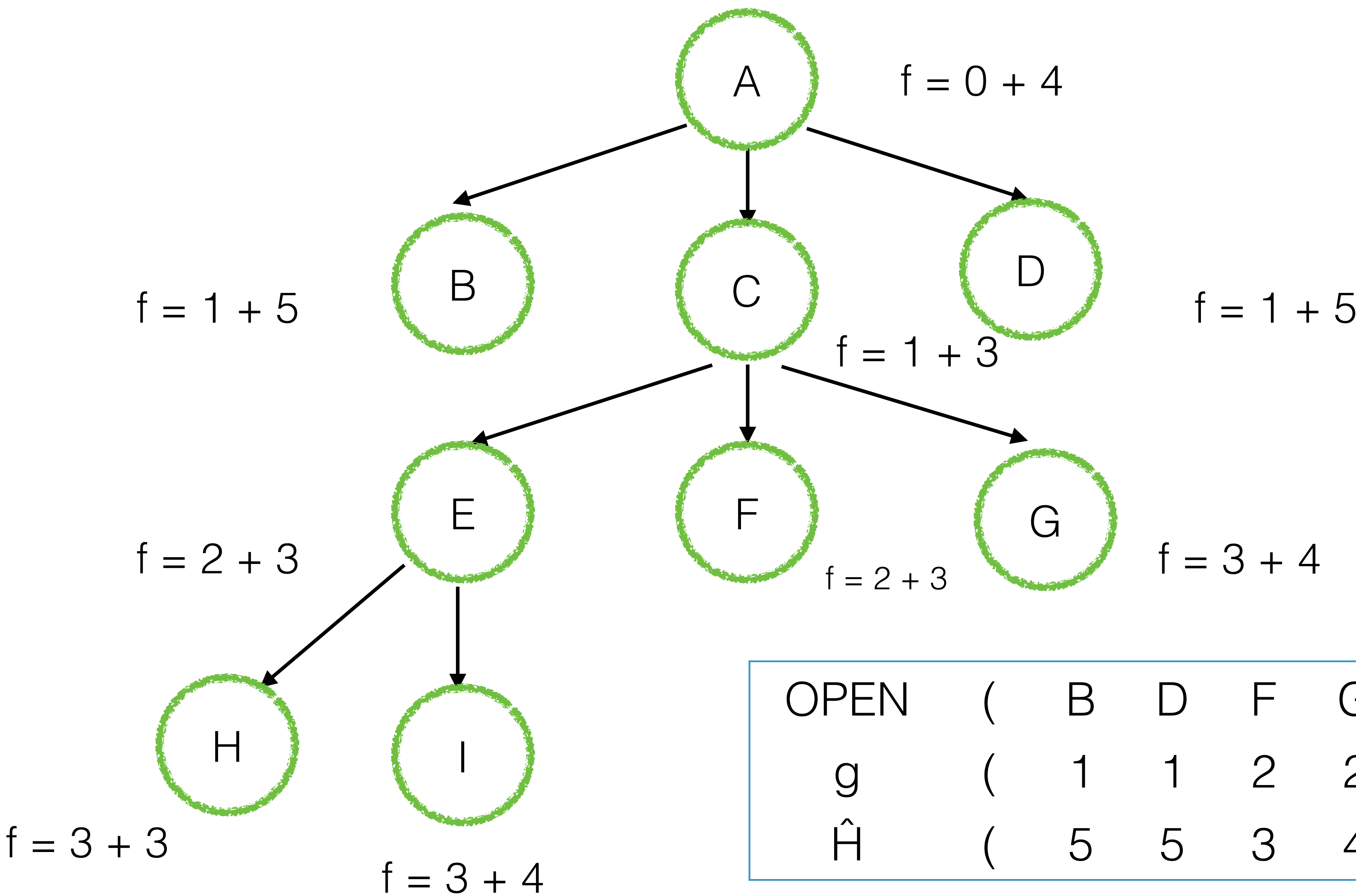


Agenda Based Search: Example



OPEN	(	B	D	F	G	H	I	)
g	(	1	1	2	2	3	3	)
$\hat{H}$	(	5	5	3	4	3	4	)

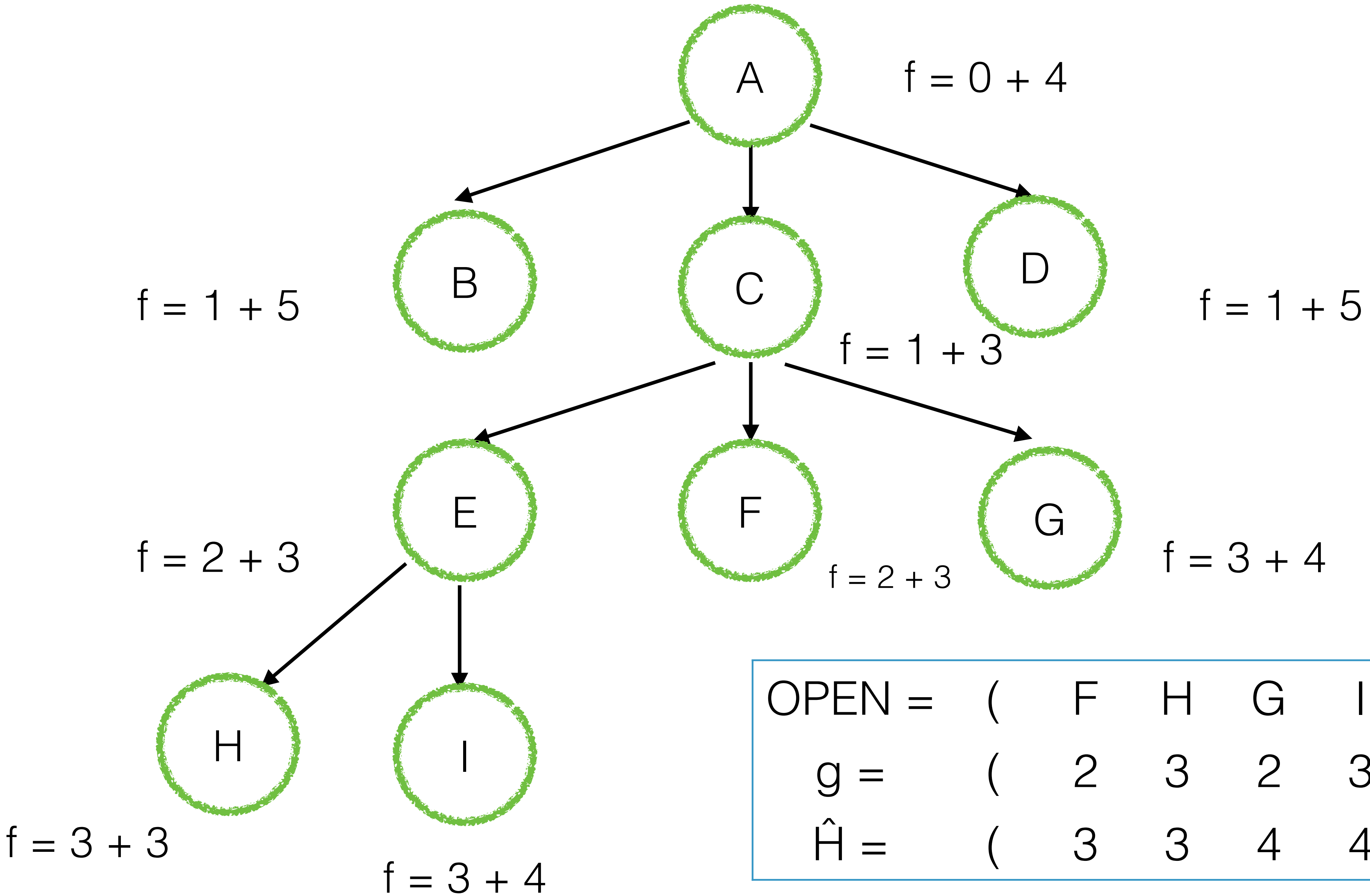
Agenda Based Search: Example



OPEN	(	B	D	F	G	H	I	)
g	(	1	1	2	2	3	3	)
$\hat{H}$	(	5	5	3	4	3	4	)

Uniform cost: reorder by g

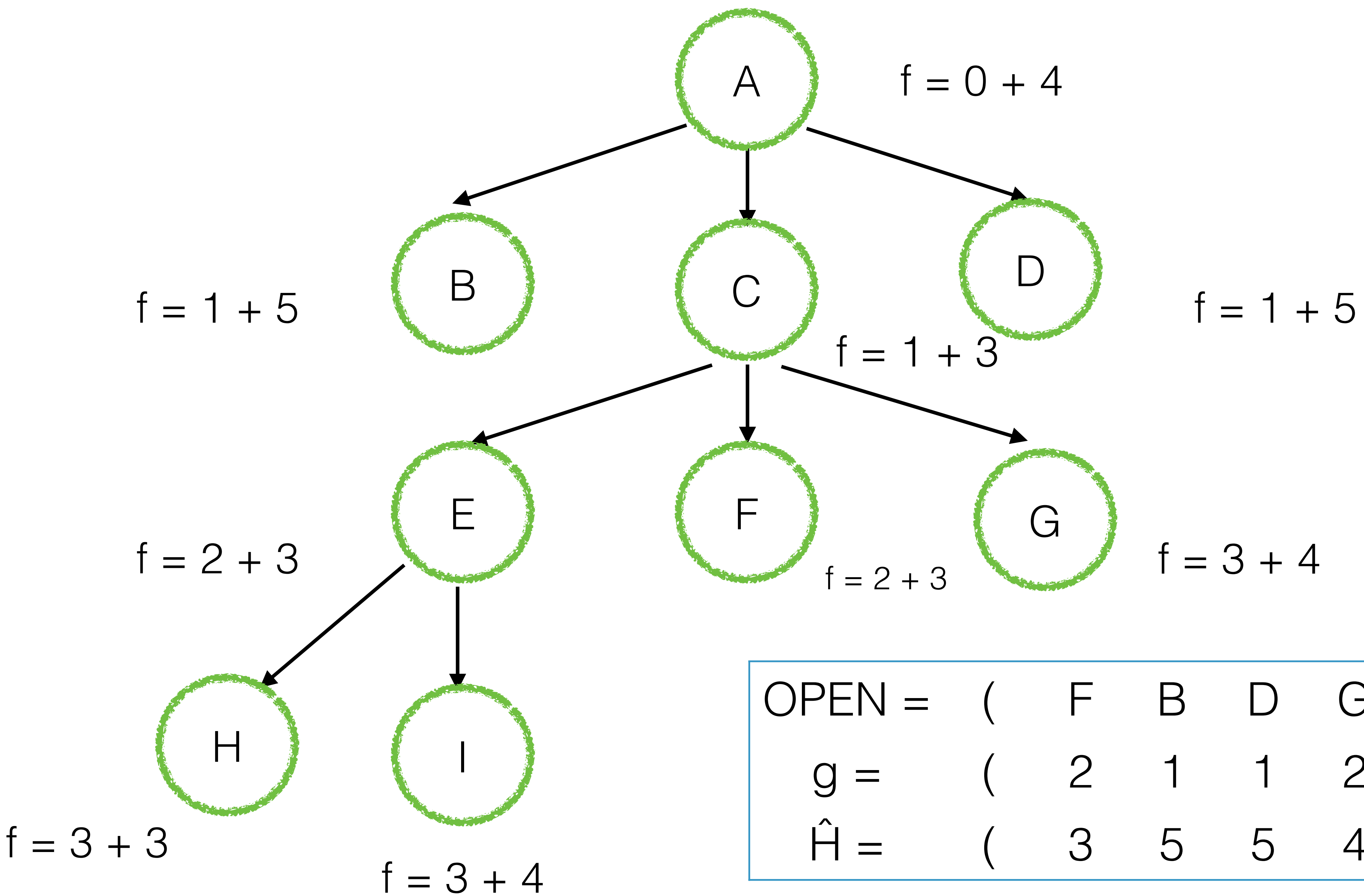
Agenda Based Search: Example



OPEN =	(	F	H	G	I	B	D	)
g =	(	2	3	2	3	1	1	)
$\hat{H}$ =	(	3	3	4	4	5	5	)

Greedy search: reorder by $\hat{H}$

Agenda Based Search: Example



OPEN =	(	F	B	D	G	H	I	)
g =	(	2	1	1	2	3	3	)
$\hat{H}$ =	(	3	5	5	4	3	4	)

A* search: reorder by $f = g + \hat{H}$

Summary

- Heuristic evaluations of cost to reach goal
- Hill climbing
- Best-first search
- A* Search
- Agenda-based search